# OPTIMIZATION, OPTIMAL DESIGN AND DE NOVO PROGRAMMING: DISCUSSION NOTES

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**Introduction**

Many older texts, with titles like *Globally Optimal Design* [1] or *Principles of Optimal Design* [2], rarely dealt with system design, its configuration or re-configuration. Rather, they focused on valuation of a given (bounded, well constrained, pre-configured) system, searching for decision or design variables that maximize a single or multiple (also given) measurement criteria or objectives.

Any system is defined by its boundaries or constraints, separating its feasible sets of alternatives, options or designs (vectors of variables) from their environment. Consequently, system design, re-design, configuration and optimization must involve a purposeful and well-directed "reshaping" of system boundaries or constraints. Simply selecting alternatives or options from an *a priori* given, pre-configured system is not design but (e)valuation. System design is a process of generation, production and creation - not just selection - of viable portfolios of alternatives.

Many engineering and system-design writers were keenly aware of the problem of system design or reconfiguration, but tended to minimize it. For example, in [2] we read:

*... one might ask to pose an optimization problem seeking the optimal configuration. This may be a noble cause, yet if we are to compare configurations, we must have a mathematical model that allows us to move from one configuration to another in our search for the optimum. We can easily recognize that each configuration will have its own set of design variables and constraints, at least, and most probably, different forms of objective function. Combining these in a single model when an optimization study will be applied is almost impossible. Some mild changes in geometric shapes can be handled by special techniques in shape optimization. Drastic changes in the design topology* <u>*cannot be dealt with mathematically*</u> [emphasis M.Z.] *in any reasonably useful way that the authors are aware of.*

Often a one-sentence disclaimer was appended, see page 20 of [2]:

*In the future discussions we will be making the tacit assumption that the models refer to single configurations arrived at through* <u>*some*</u> [emphasis M.Z.] *previous synthesis.*

Today, of course, we have to address such "almost impossible" tasks of system reconfiguration. We have to define optimization as the "shape optimization." We should openly differentiate the computational optimization from systems design. In addition, with the advances of computational power, the *corporate computation* has become a *commodity* and ceased - together with information and IT - being an effective source of competitive advantage [3, 4].

**What is Optimization?**

The notion of optimization is often constructed as a computational valuation of a *given* set of alternatives with respect to a *given* set of criteria and objectives. That is *computation* or, more precisely, *computational optimization*. Computation is different from systems design, analysis or management, i.e. from optimal design, optimization or optimality. Computation is a mathematical tool aiming for digital characterization and explication of given, fixed structures and configurations. Systems design is all about shapes, topology and configuration, much less so about computation or computational optimization.

In Fig. 1 we look at two conflicting objectives, $f_1$ and $f_2$, to be *both* identically maximized over the variable design space. The point of the picture is to show that the conflict, tradeoffs or any other forms of relationships between criteria and objectives are not inner attributes of those measures, criteria or objectives, but are outer attributes of the *objects they allege to measure*, in this case feasible sets, sets of constraints, system designs, design topologies, etc.

It is therefore apparent, that the tradeoff boundary and its shapes, like the nondominated set, Pareto-optimal solutions, efficiency frontier, productivity frontier, etc. [5], are the property of the set of options (of the objects of measurement), and not of the set of measures (of the criteria of measurements). This *is* significant because in order to truly maximize an objective function(s) one has to optimize the feasible set; the rest is valuation.

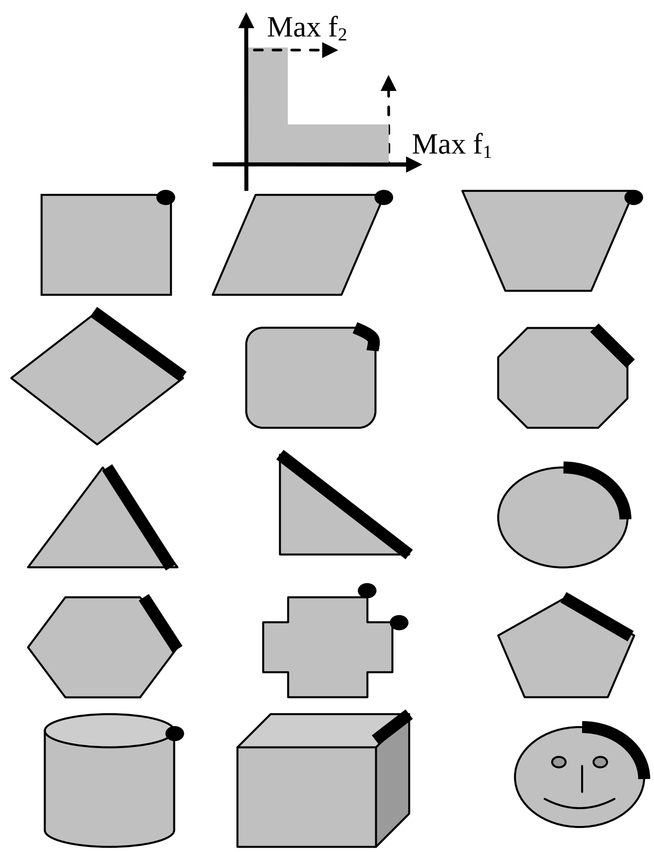


Fig. 1. Optimality and Pareto-optimal solutions are the function of the feasible set - not of the criteria or objectives themselves.

The very notion of *a priori* feasibility is therefore dubious because the purpose of system design is to expand and redefine feasibility, not to accept it axiomatically. Innovation is not about doing the same thing better, but about doing things differently and – most importantly – doing different things. In systems design it is not the efficiency but the effectiveness which is of interest. There are a few rules which have to be respected:

1. What is determined or given *a priori* cannot be subject to subsequent optimization and thus, clearly, does not need to be optimized: *it is given*.
2. What is not yet given must be selected, chosen or identified and is therefore, by definition, subject to optimization.
3. Consequently, different optimality concepts can be derived from distinctions between what is given and what is yet to be determined in problem solving, systems design or decision-making.

Traditionally, by optimal solution or optimization we implicitly understood maximizing (or minimizing) a single, pre-specified objective function (criterion) with respect to a given, fixed set of decision alternatives (situation constraints). Both the criterion and decision alternatives are given, only the (optimal) solution remains to be explicated (computed). In addition, *multiple criteria* should be explicitly considered in order to preserve the sense of practicality.

Consequently, there are at least *eight distinct optimality concepts*, all mutually irreducible, all characterized by different applications, interpretations and mathematical/computational formalisms - as displayed in Table 1.

**The Eight Concepts of Optimization**

In Table 1 we summarize the eight key optimality concepts according to a simplest classification: single versus multiple criteria against the extent of the 'given': ranging from 'all-but' to 'none except'. The traditional concept of optimality, characterized by too many 'givens' and a single criterion, naturally appears to be the most remote from a sort of optimal conditions or circumstances for problem solving as is represented by *cognitive equilibrium* (optimum) with multiple criteria[5,6].

| Number of Criteria / Given | Single | Multiple |
|---|---|---|
| Criteria & Alternatives | Traditional "Optimality" | MCDM |
| Criteria Only | Optimal Design (De Novo Programming) | Optimal Design (De Novo Programming) |
| Alternatives Only | Optimal Valuation (Limited Equilibrium) | Optimal Valuation (Limited Equilibrium) |
| "Value Complex" Only | Cognitive Equilibrium (Matching) | Cognitive Equilibrium (Matching) |

Table 1. Eight concepts of optimality: From traditional 'optimality' to cognitive equilibrium

The problems of the second row of Table 1 have already been addressed by the De Novo programming methodology, see for example in [7, 8]. Here we outline only a short LP summary.

**The De Novo Summary**

Formulate the linear programming problem:

$$\text{Max } Z = Cx \text{ s.t. } Ax - b \le 0,\ pb \le B,\ x \ge 0 \quad (1)$$

where $C \in \Re^{qxn}$ and $A \in \Re^{mxn}$ are matrices of dimensions $q \times n$ and $m \times n$, respectively, and $b \in \Re^m$ is $m$-dimensional *unknown* resource vector, $x \in \Re^n$ is $n$-dimensional vector of decision variables, $p \in \Re^m$ is the vector of the unit prices of $m$ resources, and B is the stipulated available budget.

Solving problem (1) means finding the optimal allocation of B so that the corresponding resource portfolio *b* maximizes simultaneously the values Z = Cx of the product mix *x*. Obviously, we can transform problem (1) into:

$$\text{Max } Z = Cx \text{ s.t. } Vx \le B,\ x \ge 0 \quad (2)$$

where $Z = (z_1, ..., z_q) \in \Re^q$ and $V = (V_1, ..., V_n) = pA \in \Re^n$. Let $z_{k*} = \max z_k$, k = 1, ...,q, be the optimal value for *k*th objective of Problem (2) subject to $Vx \le B$, $x \ge 0$. Let $Z^* = (z_{1*}, ..., z_{q*})$ be the *q*-objective value for the ideal system with respect to B. Then, a *metaoptimum* problem can be constructed as follows:

$$\text{Min } Vx \text{ s.t. } Cx \ge Z^*,\ x \ge 0 \quad (3)$$

Solving Problem (3) yields $x^*$, $B^*$ (= $Vx^*$) and $b^*$ (= $Ax^*$). The value $B^*$ identifies the minimum budget to achieve $Z^*$ through $x^*$ and $b^*$. Since $B^* \ge B$, the *optimum-path ratio* for achieving the ideal performance $Z^*$ for a given budget level B is defined as:

$$r^* = B/B^* \quad (4)$$

and verifies the optimal system design as (**x**, **b**, **Z**), where $\mathbf{x} = r^*x^*$, $\mathbf{b} = r^*b^*$ and $\mathbf{Z} = r^*Z^*$. The optimum-path ratio $r^*$ provides an effective and expeditious tool for an effective optimal redesign of large-scale linear systems.

### A Numerical Example

The following example has been adapted from [8, 9]:

$$\begin{aligned} \text{Max} \quad z_1 &= 50\, x_1 + 100\, x_2 + 17.5\, x_3 \\ z_2 &= 92\, x_1 + 75\, x_2 + 50\, x_3 \\ z_3 &= 25\, x_1 + 100\, x_2 + 75\, x_3 \end{aligned}$$

Subject to

$$\begin{aligned} 12\, x_1 + 17\, x_2 \phantom{+ 15\, x_3} &\le b_1 \\ 3\, x_1 + 9\, x_2 + 8\, x_3 &\le b_2 \\ 10\, x_1 + 13\, x_2 + 15\, x_3 &\le b_3 \\ 6\, x_1 + 16\, x_3 &\le b_4 \\ 12\, x_2 + 7\, x_3 &\le b_5 \end{aligned} \quad (5)$$

$$9.5\ x_1 + 9.5\ x_2 + 4\ x_3 \leq b_6$$

We assume, for simplicity, that the objective functions $z_1$, $z_2$, and $z_3$ are equally important. We are to identify the optimal resource levels of $b_1$ through $b_6$ when the current unit prices of resources are $p_1 = 0.75$, $p_2 = 0.60$, $p_3 = 0.35$, $p_4 = 0.50$, $p_5 = 1.15$ and $p_6 = 0.65$. The initial budget $B$ = \$4658.75.

We calculate $Z^* = (10916.813; 18257.933; 12174.433)$ with respect to the known $B$ (\$4658.75). The feasibility of $Z^*$ can only be secured by the metaoptimum solution $x^* = (131.341; 29.683; 78.976)$ at the cost of $B^*$ = \$6616.5631.

Because the optimal-path ratio $r^* = 4658.75/6616.5631 = 70.41$, the resulting $\mathbf{x} = (92.48; 20.90; 55.61)$ and $\mathbf{Z} = (7686.87; 12855.89; 8572.40)$. It follows that the optimal portfolio **b**, with respect to $B$ = \$4658.75, can be calculated by substituting **x** into the constraints (5). We obtain:

$$\begin{aligned} b_1 &= 1465.06 \\ b_2 &= 910.42 \\ b_3 &= 2030.65 \\ b_4 &= 1444.64 \\ b_5 &= 640.07 \\ b_6 &= 1299.55 \end{aligned} \tag{6}$$

If we spend exactly $B$ = \$4658.8825 (approx. \$4658.75) the optimum portfolio of resources to be purchased at current market prices is displayed in (6), allowing us to produce **x** and realize **Z** in criteria performance.

**Conclusion**

During this era of *commoditization* of IT, computation and information [10], it is essential to explore new sources of competitive advantage. Innovation is supplanting computation, radical changes in design topology replace the mild adjustments of habitual configurations, and discontinuous improvement upstages the continuous improvement. It is more important to do the right things than to do things right. Effectiveness moves beyond efficiency, knowledge beyond information and design beyond computation. Business model becomes the new source of competitive advantage, differentiation and collaboration reign over uniformity and competition. *Optimizing the given gives way to the designing of the optimal.*

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